Low-communication parallel quantum multi-target preimage search

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Introduction

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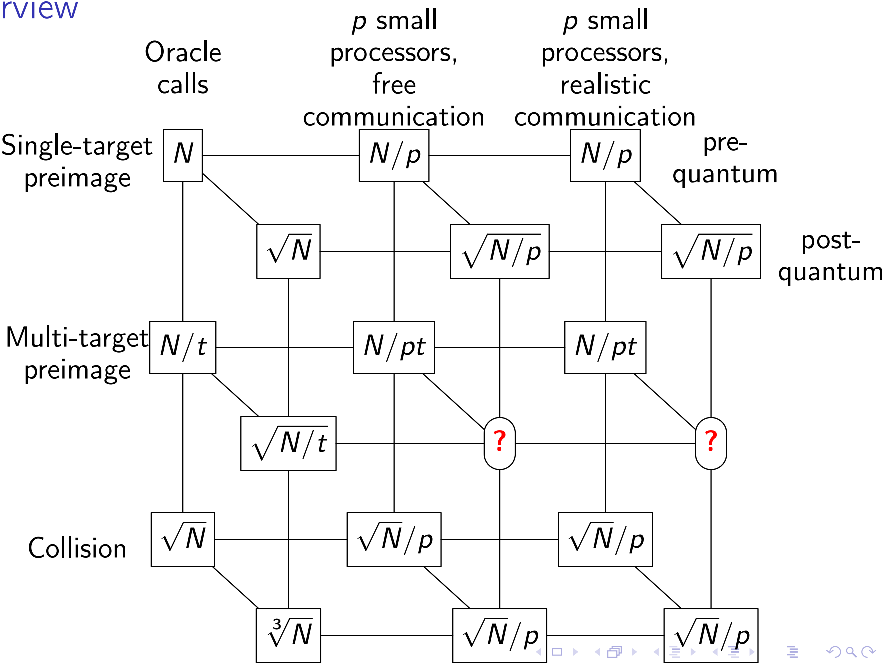
NIST has claimed that AES-128 is secure enough.

Overview



Oracle calls
p small processors, free communication
p small processors, realistic communication
pre-quantum
post-quantum
Single-target preimage
Multi-target preimage
Collision
N
N/p
N/p
$\sqrt{N}$
$\sqrt{N/p}$
$\sqrt{N/p}$
N/t
N/pt
N/pt
$\sqrt{N/t}$
?
?
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$\sqrt{N}/p$
$\sqrt{N}/p$
$\sqrt[3]{N}$
$\sqrt{N}/p$
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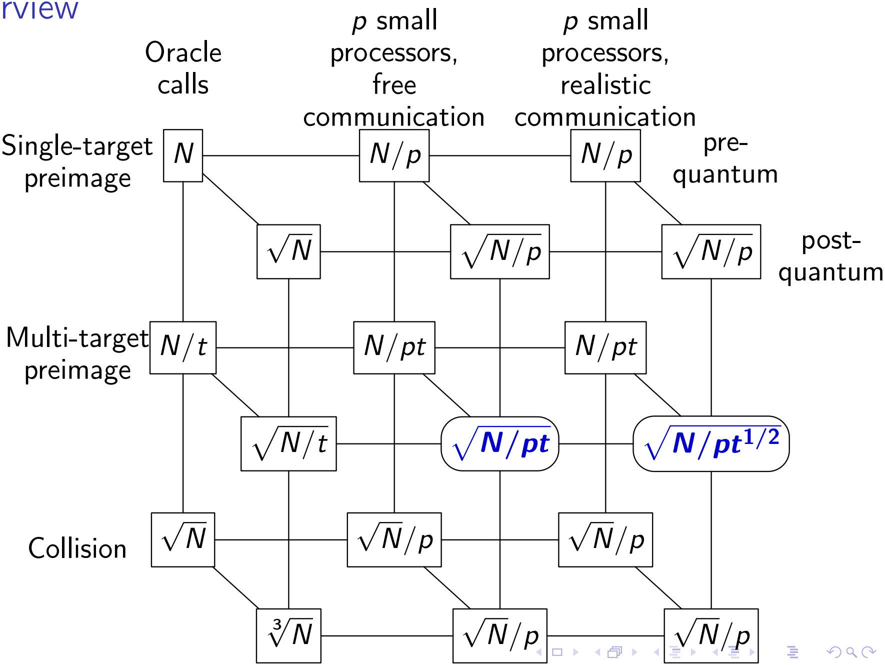
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Distinguish point

Let $H : \{0, 1\}^b$ to $\{0, 1\}^b$
Take x an input of H, $x' = H(x)$.
After take x' and apply H again, $x'' = H(x')$.
It is possible to do it n times, H^n until we satisfy a condition. In our case, we want the first $0 < d < b/2$ bits as 0.

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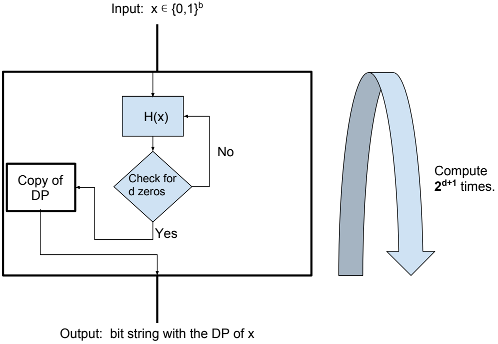
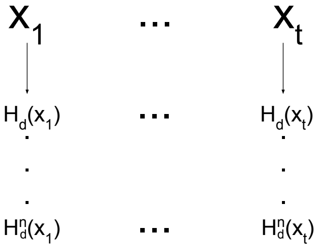
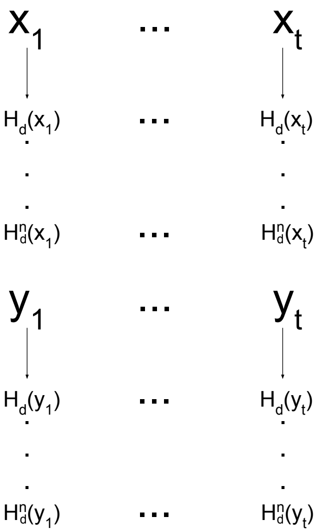



x_1
…
x_t
$H_d(x_1)$
…
$H_d(x_t)$
$H_d^n(x_1)$
…
$H_d^n(x_t)$


x1 ... xt
Hd(x1) ... Hd(xt)
Hnd(x1) ... Hnd(xt)
y1 ... yt
Hd(y1) ... Hd(yt)
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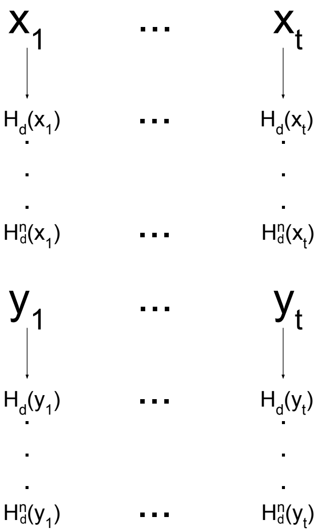

$$H_d^n(y_i) \stackrel{?}{=} H_d^n(x_j)$$

Reversibility

Reversibility of distinguish point

- Bennett-Tompa technique to build a reversible circuit for H^n;
- It is possible to achieve $a + O(b \log_2 n)$ ancillas and gate depth $O(gn^{1+\epsilon})$.

Reversibility of sorting on a mesh network

- Using the sorting strategy from "Efficient distributed quantum computing"[3];
- It is possible to perform the sorting of t elements using $O(t(b + (\log t)^2))$ ancillas and $O(t^{1/2}(\log t)^2)$ steps.

[3] Efficient distributed quantum computing
Beals, Robert and Brierley, Stephen and Gray, Oliver and Harrow, Aram W. and Kutin, Samuel and Linden, Noah and Shepherd, Dan and Stather, Mark

Finding t-images

Fix images $y_1, \ldots, y_t$. We build a reversible circuit that performs the following operations:

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- Sort the chain ends for $x_1, \ldots, x_t$ and the chain ends for $y_1, \ldots, y_t$.

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- Output 0 if a preimage was found, otherwise 1.

Example

- Imagine a function $H : \{0,1\}^{40} \rightarrow \{0,1\}^{40}$;

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- Let's say $t = 2^8$ and $p = 2^8$, for this example.
- The probability to find one preimage is roughly $t^{5/2}/N = (2^8)^{5/2}/(2^{40}) \approx 2^{-20}$;
- Each processor is going to use $\sqrt{N/pt^{3/2}}$ iterations; $\sqrt{2^{40}/2^8((2^8)^{3/2})} = \sqrt{2^{40}/2^{20}} = 2^{10}$ iterations.
- Overall we get $(2^8)^{1/4}$ speedup from attacking 2^8 targets.

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- $= \sqrt{2^{28}} = 2^{14}$ iterations.

Conclusion & What's next?

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- Circuit uses $O(a + tb + t(\log t)^2)$ ancillas;
- Depth of $O(\sqrt{N/pt^{1/2}}(gt^{\epsilon/2} + (\log t)^2 \log b))$;
- Approximately $\sqrt{N/pt^{3/2}}$ iterations.
- Created the circuit using quantum simulator for AES[4] (libquantum instead of LiQUi $|\rangle$);

[4] Applying Grover's algorithm to AES: quantum resource estimates Grassl, Markus and Langenberg, Brandon and Roetteler, Martin and Steinwandt, Rainer

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What's next?

- Check for the real number of qubits/gates;
- Is it possible to improve?

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Questions

Thank you for your attention.
Questions?
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